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A CONFUSION OF ἘΛΑΣΣΟΝΑ AND ΜΕΙΖΟΝΑ IN OUR TEXT OF PROCLUS' COMMENTARY ON THE FIRST BOOK OF EUCLID'S *ELEMENTS*

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Proclus' commentary on the first book of Euclid's *Elements* is doubtlessly the longest and most intellectually complex ancient text in the philosophy of mathematics that has been preserved. For readers inclined more toward mathematics than philosophy, the most exciting passages are probably to be found in Proclus' comments on prop. 1,27–29 of Euclid. 1,29 is the converse of 1,27 f. While many propositions in the *Elements* are converses of the ones immediately preceding them, 1,29 is the only converse whose demonstration requires a new postulate – the fifth one, which is not used by Euclid in the earlier part of the work (1,1–28). The fifth postulate states that, if a straight line, falling on two other straight lines, makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on the side on which the angles less than the two right angles are situated. The lack of evidence of this axiom – given that the Greeks were sufficiently familiar with asymptotes for curves – as well as the anomaly of the relation between 1,27 f. and 1,29 explain why mathematicians, already in antiquity, tried to prove the fifth postulate on the basis of the other postulates and axioms.

Proclus refers to Ptolemy's attempt and aptly demonstrates its failure. He himself then proposes a new demonstration – which, however, would later suffer the same fate as that of Ptolemy suffered at Proclus' own hands (which will be the case for all such 'proofs' until the 19th century, when mathematicians developed hyperbolic geometry as an equally consistent alternative to Euclidean geometry). Proclus' demonstration is circular, for he appeals to an axiom of Aristotle, which cannot be assumed unless one already presupposes the fifth postulate.

However, immediately before his own proposal Proclus discusses an objection that claimed to show that lines produced from angles less than two right angles could never meet – the contrary proposition to the fifth postulate. The argument, which is not relevant for my purpose, is analogous to Zeno's paradox about Achilles and the tortoise. As the latter paradox can be refuted by showing that both the space and the time required to pass through the space decrease continuously,[1] so the for-

1) See Aristotle, Physics 6,2 and 6,9.

mer can be rejected by grasping that, although the process of extension suggested can be repeated an infinite amount of times, there is a point of intersection of the two lines *beyond* the bound within which the continuously diminishing growth goes on. Proclus, however, does not grasp this crucial point, as both Thomas Heath and Ian Mueller rightly point out.[2]

Instead, he insists that from the hypothetical assumption that some of the lines produced from angles less than two right angles fail to intersect it does not follow that *no* such lines intersect. Using the first postulate, he simply connects two points on the two lines supposed never to meet, hereby creating an intersecting line. Needless to say this does not yet constitute a proof of the fifth postulate, the attempted demonstration for which starts only later; it only confutes the assumption that *no* such lines intersect. He then writes: εἴποι γὰρ ἄν τις ἀορίστου τῆς ἐλαττώσεως οὔσης τῶν δύο ὀρθῶν κατὰ μὲν τὴν τοσήνδε ἐλάττωσιν ἀσυμπτώτους μένειν τὰς εὐθείας, κατὰ δὲ ἄλλην τὴν ταύτης ἐλάσσονα συμπίπτειν. This is the text from the 1873 Teubner edition, authored by Gottfried Friedlein,[3] the second and last edition of the commentary to ever be published. (We owe the editio princeps to Simon Grynaeus, who published it in 1533.) Friedlein had access to fewer manuscripts than Franciscus Barocius, who produced the first translation of the work, which therefore has to be consulted by readers of Proclus' commentary. Barocius and Friedlein must have had the same text before them, since Barocius translates it in the following way: "dicat enim aliquis indefinita duorum Rectorum diminutione existente, iuxta quidem tantã diminutionem non coincidentes rectas Lineas permanere: iuxta vero aliam hac minorem, coincidere."[4] This is a faithful translation of the text that we find in Friedlein, as is the English translation by Thomas Taylor,[5] the German one by Leander Schönberger,[6] and the French one by Paul ver Eecke.[7] Only Maria

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2) The thirteen books of Euclid's Elements, translated from the text of Heiberg with introduction and commentary by Th. L. Heath, Cambridge ^{2}1926, 3 vols., I 207; I. Mueller in: Proclus. A Commentary on the First Book of Euclid's Elements, Translated with Introduction and Notes by G. R. Morrow, Princeton 1970, 290 Anm. 26.

3) Procli Diadochi in primum Euclidis Elementorum librum commentarii, rec. G. Friedlein, Lipsiae 1873, 371,7–10.

4) Procli Diadochi Lycii philosophi Platonici ac mathematici probatissimi in primum Euclidis Elementorum librum commentariorum ad universam mathematicam disciplinam principium eruditionis tradentium libri IIII, Patavii 1560, 223.

5) "For it may be said, that when the diminution of two right lines (sic!) is indefinite, the lines will remain non-coincident according to such a diminution: but will coincide according to another less than this" (The Philosophical and Mathematical Commentaries of Proclus, on the first book of Euclid's Elements . . ., in two volumes, vol. II, London 1792, 158).

6) "Denn es könnte jemand behaupten, da die Verkleinerung der 2 Rechten ohne Grenzen sei, so blieben die Geraden bis zu diesem Grade der Verkleinerung Asymptoten, bei einem anderen geringeren Grade träfen sie hingegen zusammen" (Proklus Diadochus 410–485, Kommentar zum ersten Buch von Euklids "Elementen", Halle / Saale 1945, 424 f.).

7) "Car on dira peut-être que la diminution des deux angles droits étant indéfinie, les droites restent asymptotes par telle grandeur de cette diminution et se rencontrent par une diminution différente plus petite que celle-là" (Proclus de Lycie, Les commentaires sur le premier livre des Éléments d'Euclide, Bruges 1948, 317).

Timpanaro Cardini translates it differently; she is aware of Glenn Morrow's translation, about which I will speak soon.[8] But she does not bother to tell us that for her translation she needs another text. Other translations of the whole commentary do not exist.

What does Proclus want to say? He claims that under the assumption that he is now considering as an alternative to the theory that no convergent lines intersect (and that he later pretends to confute by 'proving' the fifth postulate) some of the lines produced from angles less than two rights do not meet, but that after a certain degree of further diminution of the angles an intersection d o e s occur. This is a powerful mathematical idea, and the τις may suggest that the idea is not by Proclus himself but taken over (and adapted in order to reject the paradox Proclus is struggling with) from a mathematical text dealing with the problem of parallels.[9] The main reason for this hypothesis is that in the course of Proclus' argument the defense of this intermediate position is utterly superfluous. If, as he claims, the fifth postulate can be proven (and is thus no longer a postulate but a theorem), all lines produced from angles less than two right angles must meet – why should he then care to show that some will meet, particulary since this concession is not at all a lemma for the 'proof' that he offers? The only possible alternative explanation would be to say that Proclus was not so sure about this 'proof' and wanted to deliver at least a partial result that was not relying on Aristotle's axiom. However, I cannot detect any trace of self-doubt concerning Proclus' own attempted proof. But whatever the origin of this conception, the eminent historian of Greek mathematics Heath affirms that it contains "the germ of such an idea as that worked out by Lobachewsky, namely that the straight lines issuing from a point in a plane can be divided with reference to a straight line lying in that plane into two classes, 'secant' and 'non-secant,' and that we may define as *parallel* the two straight lines which divide the secant from the non-secant class."[10] Clearly, Proclus wants to reject the idea, for he believes he can prove the fifth postulate. However, seriously considering what one regards as counterfactual has often enough proven an important path to scientific progress. But whatever the mathematical purport and later development of this idea of Proclus, it is clear that the diminution of the angles must i n c r e a s e if we have to move from non-secant to secant lines. How then could Proclus call it "less"? While Barocius, Taylor, Schönberger, and ver Eecke did not stumble over the text, Morrow understood that there is a problem here. (Heath had translated "with an amount of lessening in excess of this", but he did not alert his readers to the fact that his translation cannot be based on Friedlein's text.)

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8) "Perché si potrà dire che, essendo indefinita la diminuzione dei due angoli retti, le rette restano asintote nella misura di una certa grandezza di diminuzione, ma per un'ulteriore diminuzione s'incontrano" (Proclo, Commento al I libro degli *Elementi* di Euclide, Pisa 1978, 296).

9) This becomes more plausible if we assume that Aristotle had already grasped the logical possibility of non-Euclidean geometries (which is not to say that he affirmed their truth). See I. Tóth, Fragmente und Spuren nichteuklidischer Geometrie bei Aristoteles, Berlin 2010. In my preface to the book I explain why I agree with most tenets of this controversial book.

10) Heath (n. 2 above) I 207. One can point to § 93, the first in the seventh chapter, of Lobachevsky's Новыя начала геометрiи сь полной теорiей параллельныхъ of 1835–38.

Morrow translates the passage in the following way: “Since ‘less than two right angles’ is indeterminate, one could say that with such-and-such an amount of lessening the straight lines remain nonsecant, whereas with another amount less than this they meet.” He then adds the footnote: “ἐλάσσονα, if it is not a slip on the part of the author, or the corruption of an original ἐλάττωσιν, must be taken proleptically, i. e. ‘another amount that still further decreases the angles.’”[11] It is certainly true that the angles have to become less, not the diminution of the angles.[12] But I do not think that Morrow’s suggested ἐλάττωσιν is a good conjecture. First, this noun is superfluous, since it is implied by the earlier part of the sentence, and second, the quantitative determination τοσήνδε seems to require more than the mere qualitative ἄλλην. In fact, it would be even misleading if τοσήνδε referred to the specific degree of diminution of the angles that constitute the parallel and not to all the diminutions that lead up to the one constituting the parallel. Rather the text should read κατὰ δὲ ἄλλην τὴν ταύτης μείζονα. But in order to be plausible, a conjecture cannot simply constitute a text that makes sense; it should also explain how the text was corrupted. This is easily done. For on the one hand, ἐλάττωσιν, used ten words before, was still operating in the mind of the scribe, and, on the other hand, if he followed Proclus’ train of thought, then he must have expected lesser angles. Therefore, he could easily have neglected that it was the process of getting at the result that was being qualified, not the result itself.

Of course one could object that such an error, caused by both a linguistic attraction and semantic considerations, might have occurred to Proclus himself.[13] It is true that we cannot exclude this possibility, for even great minds misspeak and miswrite. When we have an autograph of an author, we have to reproduce all its errors, possibly commenting in notes upon them. However, since we do not have autographs of great ancient authors, it is a sound maxim to ascribe isolated errors to scribes rather than to the authors themselves. For it is more likely that errors are produced by less gifted people.[14]

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11) Morrow (n. 2 above) 291 n. 27.

12) An anonymous reviewer suggested that ἐλάττωσιν here might have a static, not a dynamic meaning – “being less (than 180°)”. But even if this meaning could be present in 316,17 and 427,8 of the Friedlein edition (not in the other passages where the word occurs that are listed in Friedlein’s “Index rerum et verborum”), the context entails that the term in the passage at stake has a dynamical meaning (suggested in any case as the normal meaning by the word formation) – for the line is moved until it comes to the intersection. “A smaller being-smaller” would be a very strange way of expressing oneself, and I do not know any example for such an expression in the Greek language.

13) Concerning a different passage, Friedlein himself writes: “Dubium tamen est utrum lapsus calami autori an scribae sit tribuendus” (59).

14) See H.-G. Gadamer, Wahrheit und Methode, Tübingen ^{4}1975, 278: “So machen wir denn diese Voraussetzung der Vollkommenheit immer, wenn wir einen Text lesen, und erst wenn diese Voraussetzung sich als unzureichend erweist, d. h. der Text nicht verständlich wird, zweifeln wir an der Überlieferung und suchen zu erraten, wie sie zu heilen ist.”