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A TEXTUAL PROBLEM IN ARCHIMEDES, ARENARIUS 218,14 HEIBERG¹

Ἄρισταρχος δὲ ὁ Σάμιος (...) ὑποτίθεται (γὰρ) τὰ μὲν ἀπλανέα τῶν ἄστρον καὶ τὸν ἄλιον μένειν ἀκίνητον, τὰν δὲ γᾶν περιφέρεσθαι περὶ τὸν ἄλιον κατὰ κύκλου περιφέρειαν, ὅς ἐστιν ἐν μέσῳ τῷ δρόμῳ κείμενος, τὰν δὲ τῶν ἀπλανέων ἄστρον σφαιρίαν περὶ τὸ αὐτὸ κέντρον {τῷ ἀλίῳ} κειμέναν τῷ μεγέθει ταλικάυταν εἶμεν, ὥστε τὸν κύκλον, καθ' ὃν τὰν γᾶν ὑποτίθεται περιφέρεσθαι, τοιαύταν ἔχειν ἀναλογίαν ποτὶ τὰν τῶν ἀπλανέων ἀποστασίαν, οἷαν ἔχει τὸ κέντρον τᾶς σφαιράς ποτὶ τὰν ἐπιφάνειαν.

His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface. (trans. Heath)

The treatise Ψαμμίτης (Arenarius, “Sand-Reckoner”), which is attributed to Archimedes², deals with numbers sufficiently great to indicate the number of grains of sand with which the universe could be filled. The text contains a comparatively full account of Aristarchus’ heliocentric view of the universe.³ In spite of the fact that Archimedes wrote the treatise for the benefit of king Gelo, who was anything

1) The subject of this note was suggested to me by the bachelor thesis of my student Paulien Out. I thank her for drawing my attention to this text and for discussing the matter with me. Prof. J. Hogendijk (University of Utrecht) has kindly commented on an earlier draft of this note.

2) Greek text in Heiberg/Stamatis; Dijksterhuis; Mugler. I will refer to Heiberg/Stamatis. The authenticity of the *Arenarius* is questioned by Erhardt/Erhardt 580. For the date of the *Arenarius*, see Knorr 234–238.

3) Heath’s monumental study of Aristarchus has not yet been superseded. For the discussion of Aristarchus’ heliocentrism, see Heath 301–310. See also Christianidis a.o. passim. Modern detailed comments on the *Arenarius* in general, and on the passage dealing with Aristarchus in particular, are regrettably lacking. – A collection of texts (in English translation) relevant to Greek heliocentrism in general can be found in Stamatis.

but a specialist in mathematical and astronomical matters, the passage remains obscure in many respects.

Far from pretending to give an over all interpretation of the difficult passage on Aristarchus' heliocentrism, I would like to draw attention to a place where the text appears to be corrupt, to wit the phrase *περὶ τὸ αὐτὸ κέντρον τῷ ἄλιῳ κειμένον*. Archimedes' account of Aristarchus' theory starts with two plain hypotheses: 1. the sun and the fixed stars remain unmoved; 2. the earth moves in an orbit with the sun as its centre. Then he goes on to speak about the sphere of the fixed stars. According to the transmitted text this sphere is *περὶ τὸ αὐτὸ κέντρον τῷ ἄλιῳ κειμένον*. Literal translations of this phrase include "situated about the same centre as the sun" (Heath 302, Dijksterhuis 363 and Christianidis a. o. 153), "lying with the sun round the same centre" (Dreyer 137), "La sphère des étoiles inerrantes est décrite autour du même centre que le Soleil" (Duhem 420) and "qui s'étend autour du même centre que le soleil" (Mugler). Erhardt/Erhardt 579 give "the sphere of the fixed stars is concentric with the sun", which is equivocal, as will be illustrated below. Finally, Heiberg translates "circum idem centrum positam, circum quod sol moveatur", which leads to unsurmountable problems, as I will show.

But what does this phrase mean? "The sphere of the fixed stars is situated about the same centre as the sun" is most naturally taken to indicate that there is an independent point that constitutes the centre of the sphere of the fixed stars and of the sun. The first part of this phrase is clear enough, but in what sense can a given point be said to be the "centre of the sun"? A point can only be called the centre of either a sphere (or, in two dimensions, a circle) or of the circumferential motion of an object; but it is impossible to call a given point the centre of a motionless object, if this given point lies outside the object.⁴ This difficulty is obscured in most translations. Heiberg's translation, on the other hand, shows that he realized this consequence. But the price he has to pay is far too high: how in the world can one speak about a centre "around which the sun moves", when it has just been stated explicitly that the sun is motionless?⁵

There is still another difficulty. If it is assumed that the sphere of the fixed stars is situated about the same centre as the sun (whether the sun moves around this centre or stands still, is irrelevant in this respect), the consequence is that the sun is no longer the centre of the universe. And this leads to a view of the cosmos

4) The following instance may serve to explain what I mean. Madrid can adequately be designated as the centre of Spain, by which we mean that its frontier-line, which does not move, has Madrid for its centre. The city of Toledo, on the other hand, could be said too to have Madrid as its centre, if it were moving in an orbit around Madrid; but now that Toledo is firmly rooted in the Spanish soil, it would be absurd to state this. Read the sphere of the fixed stars for Spain's frontier-line, the sun for Toledo, and an unnamed point for Madrid: this shows that the sphere of the fixed stars can be said to have this unnamed point as its centre, but that the sun cannot. By the same token, the sphere of the fixed stars and the sun cannot be called concentric. – The interpretation that the centre of the sun itself is meant, will be dealt with below.

5) In fact, this interpretation would give an adequate picture of the geocentric view of the universe: in this view the sphere of the fixed stars and the sun, both of which are in a constant circular movement, are situated around the same centre, namely the earth.

which is absolutely incompatible with everything we know about the view of the cosmos held by the Ancients, because it denies the very concept of heliocentrism.⁶ We would have to assume that the centre of the universe is constituted by an unnamed point, about which nothing is stated whatsoever. Around this centre, then, there is the sphere of the fixed stars in the first place, and the sun in the second. The sun, in its turn, while not being the centre of the universe, is the centre of the orbit of the earth. And so, if Heiberg's unacceptable interpretation is accepted, this would imply that the earth moves in two ways: around the sun and (together with the sun itself) around the centre of the universe.

A third argument against this interpretation is furnished by what is stated about the distance of the orbit of the earth to the sphere of the fixed stars. The orbit of the earth has an "analogy" (ἀναλογία) to the "distance" (ἀποστασία) of the sphere of the fixed stars which is equivalent to the distance of the centre of the sphere to its surface. These last words are problematic in the context of the whole passage on Aristarchus, but for our purpose it is sufficiently clear that the distance of the orbit of the earth to the sphere of the fixed stars is constant and not subjected to alterations; now this is only possible if the sphere of the fixed stars and the orbit of the earth are concentric. And if the earth moves in a circle around the sun, while the sun is not the centre of the universe, the distance between the orbit of the earth and the sphere of fixed stars changes constantly.

In order to avoid these unacceptable consequences, there might seem to be another way of interpreting our phrase, which may be intended by Erhardt/Erhardt. Their translation, "the sphere of the fixed stars is concentric with the sun" might be taken to mean that both are situated around the centre of the sun. The same interpretation is held by Duhem 421, who summarizes Archimedes' account as follows: "La fixité absolue de la sphère des étoiles fixes; La fixité absolue du Soleil dont le centre coïncide avec le centre de cette sphère; Le mouvement annuel de la Terre sur une circonférence de cercle ayant pour centre le centre du Soleil". Now I do not want to deny the validity of this statement as such within the heliocentric view of the universe, but I only want to point out that this is not what the transmitted Greek text means. First, *περὶ τὸ αὐτὸ κέντρον τῷ ἁλίῳ κειμέναν* is the normal abbreviated way of saying *περὶ τὸ αὐτὸ κέντρον κειμέναν περὶ ὃ ὁ ἥλιος κεῖται*, "situated around the same centre around which the sun is situated". Now it is possible to say that an object has a centre (*ἔχει κέντρον*); but it is impossible to say that an object is situated around its own centre (*περὶ τὸ αὐτοῦ κέντρον κεῖται*). The phrase *ὁ ἥλιος κεῖται* states something about the sun as a whole; and the centre of the sun obviously belongs to the whole of the sun. If Aristarchus had wished to make a statement about the centre of the sun in relation to other parts of the sun, he would not have spoken just about "the sun", but about "the surface of the sun" (compare the phrase *οἷαν ἔχει τὸ κέντρον τῆς σφαιράς ποτὶ τὴν ἐπιφάνειαν* in 218,18).⁷ And if he had wished to state what Erhardt/Erhardt and Duhem may believe him to say, he might have employed a phrase like *τὰν τῶν ἀπλανέων ἀστρῶν σφαιρᾶν περὶ τὸ τοῦ ἁλίου κέντρον κειμέναν*.

6) Cf. also Plut. Plac. 2,15 (= Mor. 889B) *Τῶν μαθηματικῶν τινὲς μὲν ὡς Πλάτων, τινὲς δὲ μέσον πάντων τὸν ἥλιον*.

7) That is to say, it does make sense to state that "the sphere of the fixed stars is concentric with the surface of the sun", situated around the centre of the sun.

But, still more important, the wider context shows that Aristarchus, in the passage quoted by Archimedes, does not use the phrase τὸ τῆς σφαιρας κέντρον in the mathematical sense of “centre of a circle”, which, being a σημεῖον, has no dimensions (Eucl. I, def. 1.15.16), but in the same loose sense in which we speak of the “centre of the city”. This becomes clear in 218,18ff., where Archimedes comments on Aristarchus’ statement ὥστε τὸν κύκλον, καθ’ ὃν τὰν γὰν ὑποτίθεται περιφέρεισθαι, τοιαύταν ἔχειν ἀναλογίαν ποτὶ τὰν τῶν ἀπλανέων ἀποστασίαν, οἷα ν ἔχει τὸ κέντρον τᾶς σφαιρας ποτὶ τὰν ἐπιφάνειαν, “(such a proportion) as the centre of the sphere bears to its surface” (trans. Heath): according to Archimedes, this is impossible, because there is no analogy whatsoever between the κέντρον, which has no dimensions and therefore amounts to zero, and something else. Archimedes goes on to explain that Aristarchus employs the word κέντρον in a loose sense: ἐκδεκτέον δὲ τὸν Ἀρίσταρχον διανοεῖσθαι τὸδε· ἐπειδὴ τὰν γὰν ὑπολαμβάνομεν ὥσπερ εἶμεν τὸ κέντρον τοῦ κόσμου (...); by the word κέντρον Aristarchus means “object located in the centre”, which is for all practical intents and purposes equivalent to the preceding phrase ὅς ἐστιν ἐν μέσῳ τῷ δρόμῳ κείμενος. Just as in the current view the earth as a whole is, as it were, the centre of the universe, in Aristarchus’ view this term applies to the sun. Aristarchus is dealing with an astronomical problem (the relative position of the sun, the earth and the sphere of the fixed stars), not with a mathematical problem (the definition of the circle and its centre).⁸ Thus the word κέντρον applies to the sun as a whole, not to the mathematical centre of the sun, which has no dimensions. And this is fatal to the interpretation that Aristarchus speaks about the sphere of the fixed stars and the sun both being situated around the centre of the sun.⁹

There is still another consideration which raises suspicion against τῷ ἀλίῳ, namely the composition of the passage. In the immediate sequel (218,15–18, see above) Archimedes goes on to talk about the relation of the sphere of the fixed stars and the orbit of the earth. Therefore the comparison of the sphere of the fixed stars and the sun, as we read it in the transmitted text, is out of place. What is necessary is a comparison between the sphere of the fixed stars and the earth.

My proposal to remove the problems mentioned above is to delete the words τῷ ἀλίῳ. On this reading, the passage runs as follows: “that the sphere of the fixed stars, situated about the same centre, is so great that etc.” What is meant by “the same centre” is clear from what precedes: Aristarchus has just spoken about the

8) Cf. once more the fragment from Plutarch, quoted in note 6, μέσον πάντων τὸν ἥλιον. Cf. Christianidis a.o. 155.

9) If Archimedes had wished to state in his own words what may have been the interpretation held by Duhem and by Erhardt / Erhardt, and not in Aristarchus’ words, he would probably have done so by using the formulation he had employed a few lines before, when speaking about the traditional view of the universe (218,1–3): κατέχεις δέ, διότι καλεῖται κόσμος ὑπὸ μὲν τῶν πλείστον ἀστρολόγων ἡ σφαῖρα, ἧς ἐστὶ κέντρον μὲν τὸ τᾶς γᾶς κέντρον: “you know that the majority of astronomers give the name ‘universe’ to the sphere of which the centre is constituted by the centre of the earth”. By analogy, Archimedes might have written in his own words τὰν τῶν ἀπλανέων ἄστρων σφαῖραν ἧς ἐστὶ κέντρον τὸ τοῦ ἀλίου κέντρον. – There is no saying in how far Aristarchus’ text has been adapted by Archimedes. That the quotation is not quite literal, is already clear from the fact that it is in Dorian, which is not the dialect used by Aristarchus.

sun's being the centre of the orbit of the earth; when he states that the sphere of the fixed stars has the same centre, then, this means that it has the same centre as the earth, namely the sun. In this way everything falls in place: the sun is the centre of the universe, and the sphere of the fixed stars and the orbit of the earth are concentric, with the sun as their centre.

It remains for me to explain the intrusion of the spurious words τῷ ἁλίῳ. Several explanations offer themselves. In the first place, the words may have been added by a scribe who still clung to the geocentric view of the universe, in which the sphere of the fixed stars and the sun indeed do have the same centre, namely the earth (cf. note 5). It is, however, more probable that τῷ ἁλίῳ is the result of an original gloss.¹⁰ As we have seen above, Aristarchus does not specify what he means by “the same centre”, because it is self-evident from what precedes. But a scribe may have felt that there might be room for doubt, which induced him to add the gloss τὸν ἅλιον το τὸ αὐτὸ κέντρον.¹¹ Subsequently, τὸν ἅλιον became incorporated in the text, as happens so often with glosses. Next, τὸν ἅλιον was changed, consciously or unconsciously, into τῷ ἁλίῳ, in order to provide a complement to the immediately preceding τὸ αὐτὸ κέντρον. A third possibility, which I do not regard as very likely, is that Aristarchus himself wrote περὶ τὸ αὐτὸ κέντρον τὸν ἅλιον, with τὸν ἅλιον as an apposition to τὸ αὐτὸ κέντρον.

10) A glance at the editions by Heiberg and Mugler shows that there are many interpolations in the text of Archimedes, according to the editors. Some of these are introduced by the word *τούτέστιν*; see for instance II 154,24 τὰς γὰρ διὰ τοῦ Χ εὐθείας παρὰ τὰν ΑΔ ἀγομένας ἐν τῷ ἐπιπέδῳ ἐπὶ ταῦτά πάντα ἐντί [τούτέστιν ἐπὶ θάτερον μέρος]. In other cases an addition specifies (a part of) a mathematical figure; see for instance I 40,18–42,4 καὶ ἐπεὶ αἱ ΑΕ, ΕΒ τὰς ΑΒ [διαμέτρου] μείζους εἰσὶν (...) καὶ ἐπεὶ ἡ ἀποτεμνομένη κυλινδρική ἐπιφάνεια ὑπὸ τῶν ΑΓ, ΒΔ εὐθειῶν καὶ τὰ ΑΕΒ, ΓΖΔ [τρίγωνα] πέρασ ἔχει (...) καὶ τὰ ΑΕΒ, ΓΖΔ [ἐπίπεδα] πέρασ ἔχει (...), where Heiberg comments: “διαμέτρου, lin. 18, per se falsum sed ad figuram codicum accommodatum, interpolatori tribuendum, ut lin. 28 τρίγωνα, p. 42,4 ἐπίπεδα, aequae falsa.” Another group of interpolations aims at completing a text which was judged too concise by the interpolator, as appears to have been the case in our passage; some instances: I 154,13 οὕτως ἡ ΑΛ πρὸς τὴν ἀπὸ τοῦ κέντρον [τοῦ Δ] ἐπὶ τὴν ΑΛ κάθετον ἡγμένην; II 34,1 τὰ οὖν σύμπαντα ποτιλαβόντα τὸ περιεχόμενον ὑπὸ τε τὰς Θ (...) ἐσσούνται ἴσα (...) καὶ αἰὲ τῆ [περισσῆ] κατὰ τοὺς ἐξῆς ἀριθμοὺς περισσοῦς (...); II 136,28 (...) ἦτοι μείζον ἐστὶ το ΑΒ τοῦ Γ ἢ ὥστε ἰσορροπεῖν [τῷ Γ] ἢ οὐ; II 184,29 τὰς γὰρ διὰ τοῦ Η ἀχθείσας παρὰ τὰν ΑΓ ἐπὶ τὰ αὐτὰ ἐστὶν [τῷ τμήματι], where Heiberg remarks: “τῷ τμήματι lin. 29 interpolatori tribuere quam corrigere malui”. The interpolations at II 136,28 and at II 184,29 closely resemble the interpolation in our passage, because they too constitute a complement to ἰσορροπεῖν and τὰ αὐτά in the dative. – The only passage in the *Arenarius* where the editors have detected an interpolation is II 222,31 ἀποχωρίζομενος οὖν [τοῦ κυλίνδρου] ἀπὸ τῆς ὀψιος (...).

11) It is a well-known phenomenon that glosses usually are in the same case as the words they explain. See for instance sch. S. Ant. 54 ἀρτάναισι] ἀγγόνας; 60 κράτη] τὰς βασιλείας.

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